



Accumulator-free Hough Transform for Sequence Collinear Points

Rudi Kurniawan^{1,*}, Zahrul Fuadi¹, Ramzi Adriman²

¹Department of Mechanical and Industrial Engineering, Faculty of Engineering, Universitas Syiah Kuala

²Department of Electrical and Computer Engineering, Faculty of Engineering, Universitas Syiah Kuala

*Corresponding author: kurniawan@unsyiah.ac.id

Received : April 4, 2021

Accepted : July 21, 2021

Online : August 31, 2021

Abstract – The perception, localization, and navigation of its environment are essential for autonomous mobile robots and vehicles. For that reason, a 2D Laser rangefinder sensor is used popularly in mobile robot applications to measure the origin of the robot to its surrounding objects. The measurement data generated by the sensor is transmitted to the controller, where the data is processed by one or multiple suitable algorithms in several steps to extract the desired information. Universal Hough Transform (UHT) is one of the appropriate and popular algorithms to extract the primitive geometry such as straight line, which later will be used in the further step of data processing. However, the UHT has high computational complexity and requires the so-called accumulator array, which is less suitable for real-time applications where a high speed and low complexity computation is highly demanded. In this study, an Accumulator-free Hough Transform (AfHT) is proposed to reduce the computational complexity and eliminate the need for the accumulator array. The proposed algorithm is validated using the measurement data from a 2D laser scanner and compared to the standard Hough Transform. As a result, the extracted value of AfHT shows a good agreement with that of UHT but with a significant reduction in the complexity of the computation and the need for computer memory.

Keywords: mobile robot, autonomous vehicle, line segmentation, accumulator-free hough transform.

Introduction

The autonomous mobile robot and the autonomous vehicle attract more attention from the engineers to be applied in our modern civilization to increase productivity or replace humans for working in dangerous areas (Tang *et al.*, 2019, Koh *et al.*, 2021). The advance of technology allows the necessary parts for the assembly of an autonomous machine available on the market at a reasonable price.

An autonomous robot and vehicle consist of a complex system and many mechanical and electrical parts, sensors to fulfill its application. The 2D laser scanner is one of the widely used sensors for perception, localization, and navigation of an autonomous mobile robot and vehicle, or called a mobile robot (Cang & Borenstein, 2002, Petrov *et al.*, 2019, Liao *et al.*, 2019). This sensor measures the distance from its origin to the surrounding objects within its working area. The measurement data generated by the sensor is transmitted to the computer, where the next data processing is conducted. Usually, the mobile robot is equipped with a small size and low energy consumption embedded computer to reduce the price and overcome limited energy sources (Yongguo *et al.*, 2005, Liu & Sun, 2014).

Raw measurement data from the sensor must be processed by a suitable feature extraction technique to obtain the desired information concerning the robot surrounding area. Usually, the object shape and the distance from the sensor to the object center point are the required information for the autonomous mobile robot navigation (Astua *et al.*, 2014). The object shape is identified by combining two or more primitive geometry, such as a straight line, where the primitive geometry is extracted from the raw data. After that, the matching algorithm is applied to classify the object.

The extraction of the straight line from measurement data is one of the important steps in robot navigation. In (Nguyen *et al.*, 2005), six popular algorithms to extract the line from 2D laser rangefinder for indoor mobile robotics are evaluated, where Hough Transform is one of them. Hough Transform (HT) is widely used because of its effectiveness in detecting the lines and curves from the noisy data (Yun *et al.*, 1998). However, due to its drawbacks, this algorithm has the second-lowest speed compared to another algorithm in Nguyen *et al.* (2005).

There are two general steps of HT, as follows: 1) the transformation of the points into the parameter space, 2) preparation of an accumulator array for voting and finding the highest vote. The highest vote in the accumulator array corresponds to the desired solution. However, there are two main drawbacks addressed by the researchers to the Hough Transform, as follow: 1) the precision of Hough Transform directly proportional with the number of parameter increment in the recursive calculation, 2) the need of the accumulator array including the difficulty to choose its appropriate grid size. (Mukhopadhyay & Chaudhuri, 2015, Spratling, 2016, Nguyen *et al.*, 2005, Hajjouji *et al.*, 2020).

The latest comprehensive review of Hough transform in various formulations, major variants, limitations, modifications, implementation issues, and few applications in some selected fields is presented in Mukhopadhyay & Chaudhuri (2015). The researchers propose different strategies and approaches to reduce the recursive computation to transform the points into the parameter space and the size of the accumulator array; however, the accumulator array is still necessary. Imiya *et al.* (2002) proposed the accumulator-free Hough Transform, but the computational complexity increases, which is less suitable for real-time application. Another accumulator-free Hough Transform is discussed in Yu *et al.* (2004), where the algorithm is applied for the ellipse shape. The computational complexity is the function of the number of points and the size of Hough Space, which will increase when the Hough Space grid increases. In addition, the vote from the Hough Space is necessary; even the accumulator is not necessary. Moreover, the recursive computation to scan the vote within the Hough space is time-consuming and considers increasing the complexity (Wang *et al.*, 2020, Chen *et al.*, 2021).

This paper proposes an algorithm based on Universal Hough Transform to reduce the complexity and increase computation speed. The improved algorithm is called Accumulator-free Hough Transform, which is intended for sequenced measurement data generated, for instance, from the 2D laser rangefinder.

Hough Transform

Hough Transform is one of many popular algorithms to extract the parameters of the dataset, which represent a special shape such as a straight line or circle. This algorithm is invented by Paul Hough (Hough, 1962) to identify a line in the image. The algorithm is later improved and generalized by Duda and Hart (Duda & Hart, 1972). In this paper, the generalized Hough Transform, later called Universal Hough Transform, allows the algorithm to be applied universally for straight lines, ellipses, and circles. The popularity of this algorithm increases together with the advancement of computer applications in data processing. To have the overview, the Hough Transform is explained briefly in the following.

Suppose a set of data consists of n collinear points $\{x_1, x_2, \dots, x_n\}$, which is depicted in Figure 1. a. Then, the collinear points are fitted by a suitable algorithm such as simple linear regression. The fitted line is shown in Figure 1. a. Since the collinear points have no error/noise, the line is fitted perfectly.

The fitted line has a perpendicular distance r to the origin O and angle θ to the horizontal axis. The so-called normal parameters r and θ represent a straight line in a polar coordinate system. In the mathematical equation, the normal parameters are written as

$$r = x \cos \theta + y \sin \theta \quad (1)$$

where x and y are the intersection point of the perpendicular line r , θ with the fitted line in Figure 1. a.

The UHT usually requires at least three general steps to extract the normal parameters from a dataset. In the first step, each point in Figure 1. a is transformed by (1) into sinusoidal curves on the parameter space $r - \theta$, where θ of (1) is restricted to $0 \leq \theta \leq \pi$ with a specific increment, for instance, 1 degree. According to (Hough 1962), the restriction θ mentioned previously leads to unique normal parameters. Furthermore, the normal parameters of the straight line in the x - y plane matches with a specific point on $r - \theta$ plane.

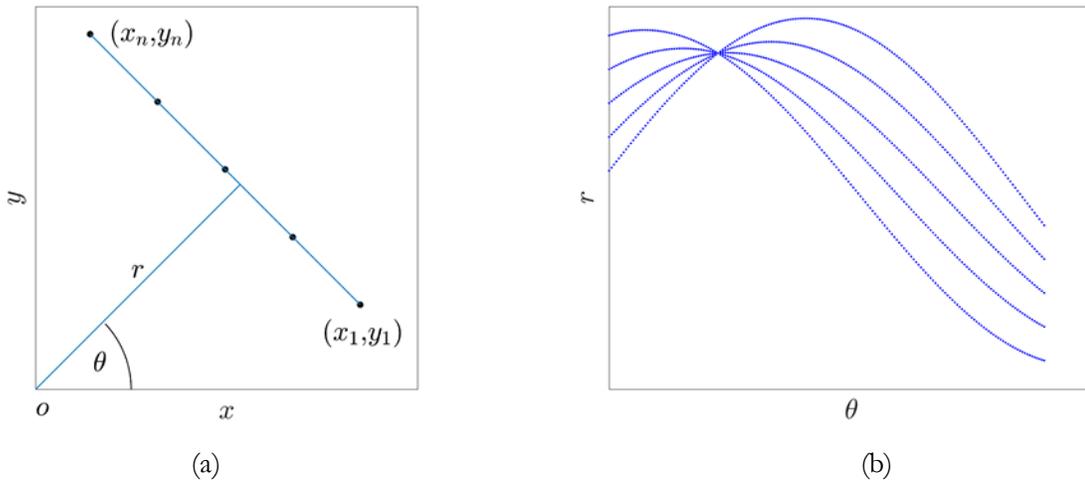


Figure 1. Collinear points (a) and its transformation (b).

The transformation of points $\{x_1, x_2, \dots, x_n\}$ in Figure 1.a into sinusoidal curves is shown in Figure 1. b. Each sinusoidal curve in $r - \theta$ plane corresponds to a point in the x-y plane. The most intersecting point of the sinusoidal curve represents the normal parameters of fitted/straight line in Figure 1. a. Since the most intersecting point can not be extracted from the $r - \theta$ plane directly in the computation, the second step is necessary.

In the second step, the so-called accumulator array is prepared (Duda & Hart, 1972). The accumulator array is a matrix, the size of which is defined by the quantization of r and θ of parameter space. It should be noted that the accumulator array is a representation of the parameter space, or in other words, a single cell in the accumulator represents a rounding of quantized r and θ . The function of the accumulator array is to store the so-called vote, which is generated during the transformation explained in the previous paragraph.

A single number of quantized θ is required by equation (1) to compute r to transform a set of data (x, y) into parameter space. Based on input value θ and output value r , the cell location within the accumulator array is determined, and a vote is given within the cell.

Figure 2 shows a small part of the accumulator near the most intersecting point, which is generated from the transformation of the collinear point in Figure 1. Since the number of points in Figure 1 is 5 and the data have no noise, then the most intersecting point is located in a single cell. In this example, the maximum vote generated by the transformation is equal to the number of collinear points, as depicted in Figure 2.

0	0	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	0	0	0
0	0	0	5	0	0
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	1	0

Figure 2. Accumulator array.

Random Sample Consensus

Before we continue to the next section, it is important to briefly overview the Random Sample Consensus (Ransac) (Fischler & Bolles, 1981). Ransac is another popular algorithm applied to estimate the straight line from the measurement data generated by a 2D laser range finder (Nguyen *et al.*, 2005). The measurement results generated by the laser range finder usually contain unnecessary data called outliers coming from many sources. To have a best-fitted line, the outlier must be removed from the dataset, and the rest of the data called inlier is fitted. This idea of Ransac is applied in this research to estimate the possible solution, which is explained in the following section.

Suppose a dataset in a cartesian coordinate system as depicted in Figure 3. Two points in the circle are chosen as the samples, and the straight line is fitted. A threshold is defined where its value depends on the need. The perpendicular distances from each measurement point to the fitted line are computed. If the distance is less equal to the threshold, then the point is considered as the inlier. To make the understanding easier, the threshold line is shown as the dashed line in Figure 3. The measurement results in between the dashed line are the inliers, and those outside it are outliers. This process is repeated for other sample combinations until the termination condition is achieved. The termination condition can be defined by the number of inliers or until all samples are combined.

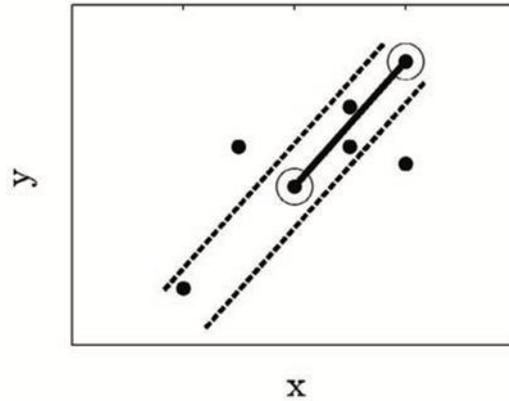


Figure 3. Random Sample Consensus (Kurniawan *et al.*, 2016).

Proposed Algorithm: Accumulator-free Hough Transform

One of the drawbacks of the UHT is the need for an accumulator array. The computational complexity and the size of the accumulator array rise proportionally to the precision level of the normal parameters. The more precision required, the more the computation complexity, including the size of the accumulator array is. However, low complexity, efficient memory use, and high precision result are the important keywords as the advantage in the real-time application, such as using a 2D laser scanner in an autonomous vehicle.

To reduce the computational complexity of the UHT, the most intersecting point has to be extracted directly from the parameter space without using an accumulator array. This is possible by direct extraction of the intersecting point of the sinusoidal curves in Figure 1.b. The computational steps of the proposed Accumulator-free Hough Transform (AfHT) algorithm are explained in the following sections.

Suppose we want to calculate the intersection of two sinusoidal curves of two sequence points (x_i, y_i) and (x_{i-1}, y_{i-1}) from n data/collinear points as depicted in Figure 1, with $i \in \{1, 2, 3, 4, \dots, n\}$. The line equations of the points of the sequence are:

$$r_{i-1} = x_{i-1} \cos \theta_{i-1} + y_{i-1} \sin \theta_{i-1} \quad (2)$$

$$r_i = x_i \cos \theta_i + y_i \sin \theta_i \quad (3)$$

At the intersection point in the parameter space, both equations have a similar value of r and θ . Then, (2) and (3) are rewritten to be

$$x_{i-1} \cos \theta_{i-1} + y_{i-1} \sin \theta_{i-1} = x_i \cos \theta_i + y_i \sin \theta_i \quad (4)$$

Then, (4) is simplified as

$$\theta = \arctan \frac{x_{i-1} - x_i}{y_i - y_{i-1}}, \quad (5)$$

The flowchart of this proposed algorithm is depicted in Figure 4. To obtain θ for all quadrants, the function *arctan2* in the programming language is recommended to avoid the ambiguous value. θ is submitted into (2) or (3) to obtain r_{i-1} or r_i . It should be noted that the singularity occurs when the denominator of (5) is zero. In that case, a method to handle the singularity is necessary.

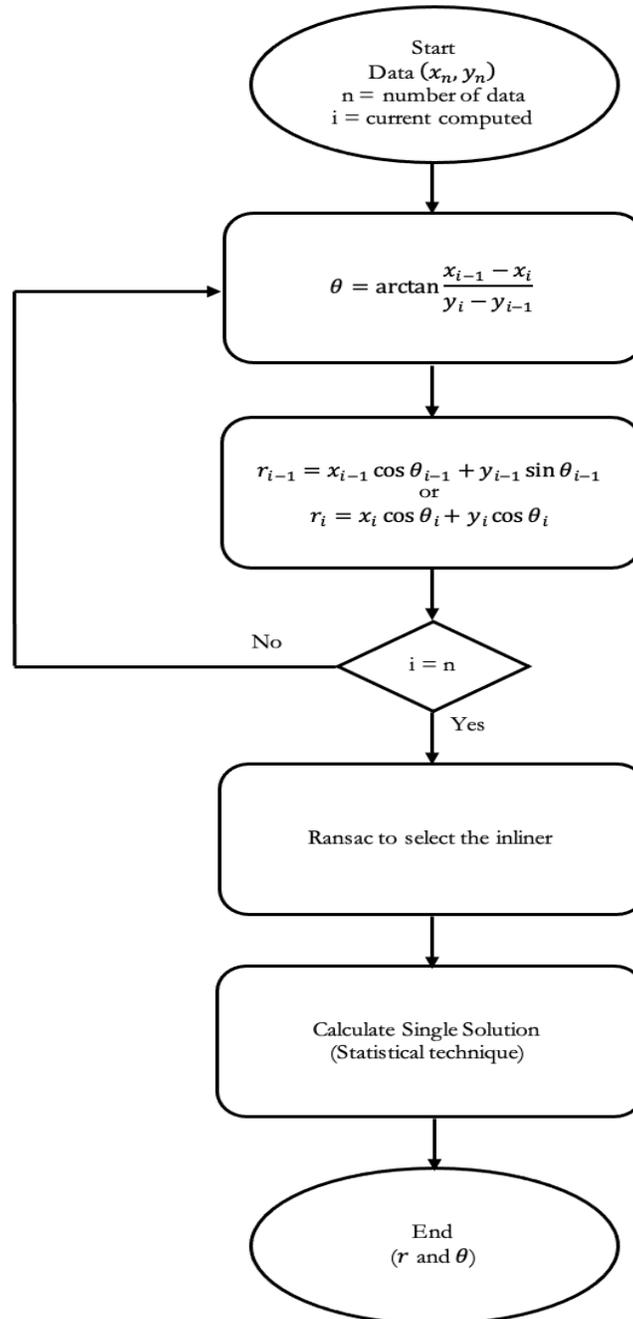


Figure 4. Flowchart of the proposed algorithm.

At this point, we already obtained the normal parameters of two collinear points (x_i, y_i) and (x_{i-1}, y_{i-1}) . If the number of points is more than two, the computation process to obtain r and θ is repeated for other sequence points combination. In this research, the sample is chosen from two sequencing points based on the assumption that the data points in measurement are in sequence.

The data in Figure 1 is generated from a straight-line equation; therefore, the points are perfectly collinear without any noise. However, the data collected in the real measurement usually contains a certain number of noises due to many reasons, such as the quality of the sensor. When noisy data is given as the input to the proposed algorithm, the computation result can be in multiple solutions. An illustration of the possible solutions from noisy data after the computation of the normal parameters using the above method is shown in Figure 5 (indicated by a circle).

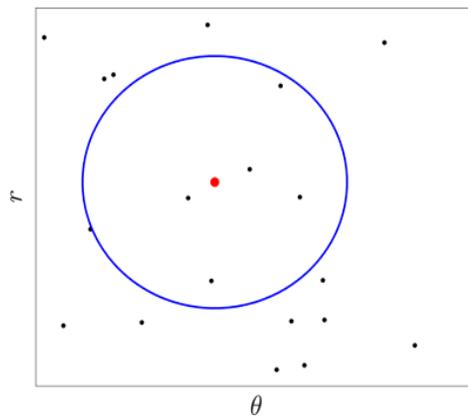


Figure 5. Modified Random Sample Consensus.

Now, Ransac is adopted to minimize the number of solutions, as explained previously. The purpose is to classify the dataset into inlier and outlier by a specific threshold which is defined according to a certain value. In the Ransac explained previously, the threshold is defined by the dashed-line. In this case, a single sample is chosen, and the threshold is defined as the radius from the sample to the circumference. The illustration of the threshold is shown by the circle in Figure 5, where the red-dot is the sample. The data inside the circle is classified as the inlier, and that outside is the outlier. To consider the sample as the possible solution, a minimum number of inliers has to be defined. Then, all samples which have the minimum number of inliers are considered as the possible solutions.

After possible solutions are identified, a further step is necessary to determine the single solution. This can be done by suitable data processing methods such as statistical techniques. In this research, the single solution is determined by averaging the values of possible solutions.

Accumulator-free Hough Transform testing

The proposed algorithm is validated using the data generated by a 2D laser scanner. Two-dimensional laser scanners such as SICK LMS series and Hokuyo scanning rangefinder are used widely in automation and autonomous vehicle navigation to sense its surrounding area. Both sensors work based on the Time-of-Flight principle to measure the distance from the laser origin to an object. A sensor rotates a mirror to reflect a laser beam radially to measure the distance to the surrounded objects. An example of measurement from SICK LMS 500 is depicted in Figure 6.a.

The dataset in Figure 6.a is collected by the sensor with an aperture angle of 180° and angular resolution of 1° . According to the manual of the selected mode, the sensor has a systematic error of 35mm and a statistical error of 9 mm. The measurement data in Figure 5.a is depicted in three different colors to give the reader a better understanding of the comparison in the next step. The same color represents the collinear points.

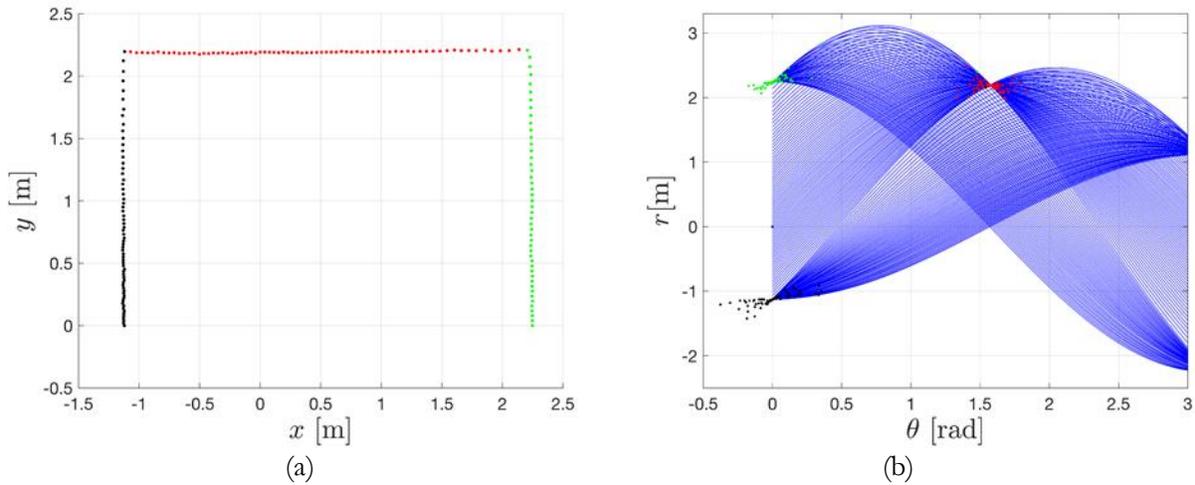


Figure 6. (a) Measurement data collected by 2D laser scanner Sick LMS 500, (b) $r - \theta$ plane of (a) measurement data

The measurement data in Figure 6. a is transformed by the proposed algorithm and shown in Figure 6. b. The dot's color in Figure 6. b corresponds to the color of collinear points in Figure 6.a. Although not necessary in this proposed algorithm, the sinusoidal curves of the universal Hough Transform are shown to compare the location of possible solutions using this proposed algorithm. Due to the figure limit size, the result of the proposed algorithm cannot be shown clearly in Figure 5.b. For this reason, the magnification near an intersection point in Figure 6.b is depicted in Figure 7 (without magenta circle).

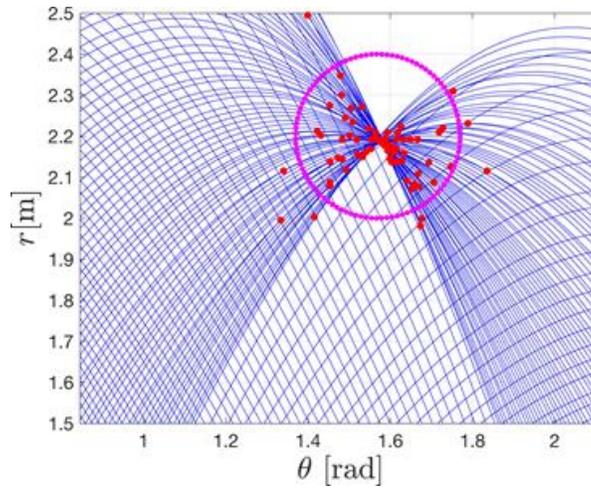


Figure 7. Accumulator array

Figure 7 shows the multiple solutions of the proposed algorithm. The solution is not unique due to the noises from the data input in Figure 6.a. To minimize the solution, the modified random sample consensus explained previously is applied. In the end, the minimized solutions are averaged to obtain the unique/single solution.

To obtain the performance of the proposed algorithm, the normal parameters of the measurement data in Figure 6. are also extracted using the UHT. The distance r is computed by (1) with $\theta = \{0, 0.0175, 0.0349, \dots, \pi\}$. According to the sinusoidal curve of the UHT shown in Figure 6.b, the accumulator

array with a size of 800 x 181 is prepared to store the votes. Each pixel of accumulator row and column represents 0.01m length and angle 0.0175 radian, respectively.

Result

The comparison results of the normal parameters computed by the UHT and AfHT is shown in Table 1. The result shows a good agreement, where the maximum difference of r at the green collinear points is 0.034m or 1.5% compared to the largest r in Tab.1. Meanwhile, θ has the maximum inequality of 0.036rad at the red collinear points, or 2.3% compared to the largest θ in Table1.

Table 1. Normal parameters comparison of Universal Hough Transform with Accumulator-free Hough Transform.

Color	$r(m)$		$\theta(rad)$	
	UHT	AfHT	UHT	AfHT
Green	2.24	2.274	0	-0.020
Red	2.185	2.168	1.579	1.615
black	-1.125	-1.105	0.008	0.018
Maximum difference		0.034		0.036
		1.5%		2.3%

Discussion

It should be noted that the size of the accumulator array of the UHT influences the value of the extracted normal parameters. On the other hand, the approach used to minimize the possible solutions and those to obtain the unique solution of this proposed algorithm also contributes to the difference, as explained in Table 2.

The complexity of the algorithm is also important to be discussed. One of the common measurable values presented by the researchers to compare the algorithm's complexity is the computational time. However, the computer processor is not always running constantly and, in the end, influences the computational time. For that reason, the size of the accumulator array and the number of calculations is chosen as the measurable references to compare the complexity of both algorithms, as shown in Table 2.

Table 2. Complexity comparison of UHT with AfHT.

Complexity	UHT	AfHT
Size accumulator array	800 × 181	0
Number of calculations	181 × 181 = 32761	181

The number of calculation UHT is given by the repetition of the calculation (1), where the number of the measurement data in Figure 5.a is 181, and θ is discretized into 181. Compared to UHT, the AfHT has only a 0.55% calculation number.

Conclusion

An AfHT for parameters extraction from the 2D sequenced measurement data has been proposed and investigated in this paper. The improvement of the UHT develops the method. In the AfHT, a mathematical model based on the UHT and the Ransac is used to remove the need to use the accumulator array. The extracted value of AfHT shows a good agreement with that of UHT but with a significant reduction in the complexity of the computation and the need for computer memory. In the real-time application where low complexity and low memory are required, the AfHT is a promising algorithm. In the future, it is recommended to apply this algorithm in a real-time application.

Acknowledgment

This research is supported by the Universitas Syiah Kuala, Banda Aceh, Indonesia, under grant number 337/UN11.2.1/PT.01.02/PNBP/2020.

References

- Astua, C., Barber, R. Crespo, J. and Jardon, A. 2014. Object detection techniques applied on mobile robot semantic navigation. *Sensors*, 14: 6734-6757. doi:10.3390/s140406734.
- Hough, P.V.C. 1962. Method and means for recognizing complex patterns, U.S. Patent 3,069,654, Dec. 18, 1962.
- Li, H., Lavin, M.A. Master, R.J.L. 1986. Fast hough transform: A hierarchical approach. *Computer Vision, Graphics, and Image Processing*, 36:139-161. doi: 10.1016/0734-189X(86)90073-3.
- Imiya A., Hada T. and Tatara K. 2002. The hough transform without the accumulators, In Caelli, T., Amin A. Duin R.P.W. de Ridder, D. Kamel M. (eds) *Structural, Syntactic, and Statistical Pattern Recognition. SSPR /SPR 2002. Lecture Notes in Computer Science*, 2396. Springer, Berlin, Heidelberg, doi: 10.1007/3-540-70659-3_87.
- Hajjouji, I.E., Mars, S. Asrih, Z. and Mourabit, A.E. 2020. A novel FPGA implementation of hough transform for straight lane detection, *Engineering Science and Technology an International Journal*, 23 (2):274-280. doi: 10.1016/j.jestch.2019.05.008.
- Wang, J., Wang, G. Cheng, L. 2020. Texture extraction of wear particles based on improved random hough transform and visual saliency, *Engineering Failure Analysis*, 109. doi: 10.1016/j.engfailanal.2019.104299.
- Chen, J., Qiang, H. Wu, J. Xu, G. and Wang, Z. 2021. Navigation path extraction for greenhouse cucumber-picking robots using the prediction-point Hough transform. *Computers and Electronics in Agriculture*, 180. doi: 10.1016/j.compag.2020.105911.
- Koh, K.H., Farhan, M. Yeung, K.P.C. Tang, D.C.H. Lau, M.P.Y. Cheung, P.K., and Lai, K.W.C. 2021. Teleoperated service robotic system for on-site surface rust removal and protection of high-rise exterior gas pipes. *Automation in Construction*, 125, 103609. doi: 10.1016/j.autcon.2021.103609.
- Liao, M., Wang, D. and Yang, H. 2019. Deploy indoor 2D laser SLAM on a raspberry Pi-Based mobile robot, 11th International Conference on Intelligent Human-Machine Systems and Cybernetics (IHMSC), 7-10, doi: 10.1109/IHMSC.2019.10097.
- Liu S. and Sun, D. 2014. Minimizing energy consumption of wheeled mobile robots via optimal motion planning, in *IEEE/ASME Transactions on Mechatronics*, 19(2): 401-411. doi: 10.1109/TMECH.2013.2241777.
- Yongguo, M., Lu, Y-H. Hu, Y.C. and Lee, C.S.G. 2005. A case study of mobile robot's energy consumption and conservation techniques, *ICAR '05. Proceedings. 12th International Conference on Advanced Robotics*. doi: 10.1109/ICAR.2005.1507454.
- Fischler, M.A., and Bolles, R.C. 1981. Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography, *Commun. ACM*, 24(6): 381-395. doi: 10.1145/358669.358692.
- Tang, M., Gu, Y. Wang, S. and Liang, Q. 2019. DCBot: An autonomous hot-line working robot for 110 kV substation, *Robotics and Autonomous Systems*, 119:247-262. doi: 10.1016/j.robot.2019.07.008.
- Petrov, P., Georgieva, V. Nikolov, S. and Mihaylova, A. 2019. Real-time laser obstacle detection system for autonomous mobile robot navigation, *X National Conference with International Participation (ELECTRONICA)*, 1-4, doi: 10.1109/ELECTRONICA.2019.8825623.
- Mukhopadhyay, P., and Chaudhuri, B.B. 2015. A survey of Hough Transform. *Pattern Recognition*, 48(3):993-1010, ISSN 0031-3203, doi: 10.1016/j.patcog.2014.08.027.
- Duda, R.O., and Hart, P.E. 1972. Use of the Hough transformation to detect lines and curves in pictures. *Communications of ACM*, 15(1).
- Kurniawan, R., Othman, M.F. Schramm, D. and Bruckmann, T. 2016. Combined use of modified hough transformation, random sample consensus and linear least square to extract the normal parameterization of a straight line: an application for cable driven parallel robots. *Procedia Technology*, 26:382-388. doi: 10.1016/j.protcy.2016.08.049.
- Spratling, M.W. 2016. A neural implementation of the hough transform and the advantages of explaining away. *Image and Vision Computing*, 52:15-24. doi: 10.1016/j.imavis.2016.05.001.

- Nguyen, V., Martinelli, A. Tomatis, N. and Siegwart, R. 2005. A comparison of line extraction algorithms using 2D laser rangefinder for indoor mobile robotics, 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, Edmonton, Alta., 1929-1934. doi: 10.1109/IROS.2005.1545234.
- Yun, X., Latt, K. and Glennon, J.S. 1998. Mobile robot localization using the Hough transform and neural networks, Proceedings of the 1998 IEEE International Symposium on Intelligent Control (ISIC) held jointly with IEEE International Symposium on Computational Intelligence in Robotics and Automation (CIRA) Intell, Gaithersburg, MD, USA, 393-400. doi: 10.1109/ISIC.1998.713694.
- Yu, X., Leong, H.W. Xu, C. and Tian, Q. 2004. A robust and accumulator-free ellipse hough transform, In Proceedings of the 12th annual ACM international conference on Multimedia (MULTIMEDIA '04). Association for Computing Machinery, New York, USA, 256–259. doi: 10.1145/1027527.1027585
- Cang, Y. and Borenstein, J. 2002. Characterization of a 2D laser scanner for mobile robot obstacle negotiation, Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No.02CH37292), 3:2512-2518. doi: 10.1109/ROBOT.2002.1013609.